ASPECTS REGARDING THE SIGNIFICANCE OF THE CURVATURE TYPES AND VALUES IN THE STUDIES OF GEOMORPHOMETRY ASSISTED BY GIS

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Abstract: The curvature of the terrestrial surface enters into the category of primary parameters, directly derived, on the strength of the altimetric values stored up in the grid type elevation structures. Geomorphological, it indicates the changing rate of the slope or of the orientation per unit length, in the XY plan. There are many types of curvatures, but in the more important morphometric analyses they are the following: profile curvature, plan curvature, tangential curvature, longitudinal curvature, transverse curvature, maximum curvature, minimum curvature, medial curvature, general curvature. This article aims at highlighting the differentiations that appear in the qualitative interpretation, in a geomorphic sense, the results of the processing curvatures’ varieties in a series of GIS type programmes. At the same time, we had as our goal, the identification of the causes for these interpretative disparities, causes that are found in the mathematical apparatus used at the elaboration of the algorithms for extracting the curvature values and in the ambiguities concerning their definition.

Key words: curvature, geomorphology, morphometry, GIS

INTRODUCTION
The automatic extraction of the morphometric parameters, on the strength of digital models of elevation, with the help of GIS type applications represents already a common work technique in the studies of geomorphology, to which development many researchers made their contribution (Evans, 1972, 1980; Zevenbergen & Thorne, 1987; Burrough & Mcdonell, 1988; Dikau,1989; Moore et al., 1991,1993; Mitasova & Hofierka,1993; Guth, 1995, 2003; Wood, 1996; Florinsky, 1998; Wilson & Gallant, 2000; Shary et al., 2002; Schmidt et al., 2003).

The curvature of the terrestrial surface enters into the category of primary parameters, directly derived, on the strength of the altimetric values stored up in the grid type elevation structures. Geomorphological, it indicates the changing rate of the slope or of the orientation per unit length, in the XY plan (Gallant & Wilson, 2000). The unit of measure is radian / meter, and in some cases the curvature values are multiplying with 100, because they are subunitary and difficult to operate with them.

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http://istgeorelint.uoradea.ro/Reviste/Anale/Anale.htm
There are many types of curvatures, established on the strength of some varied criteria, but in the more important morphometric analyses they are the following: profile curvature, plan curvature, tangential curvature, longitudinal curvature, cross-sectional curvature, maximum curvature, minimum curvature, mean curvature, general curvature.

Their description is very hard, because as Schmidt (2003) and Jenness (2012) observe, there are so many equations and definitions, some of them having a contradictory character. In what concerns the results’ interpretation, confusions can be even bigger, because in some papers and tutorial programmes it is said that the positive values of a curvature type show concave surfaces, and the negative values reflect the convex character of the surfaces, while in other treatises and programmes the interpretation is reversed.

Our study very much intends to solve these qualitative interpretation problems, problems that are traced even in the geomorphological literature from Romania, through a comparative analysis of the curvatures’ types, using a set of algorithms incorporated in more GIS type applications.

**TEST AREA AND DATA SOURCES**

The working area is situated on the North-West side of the Apuseni Mountains, being graft on the structural contact between the metamorphic formations of the Plopiș ridge and Barcău Hills’ sedimentary (figure 1), fact which offers the region a great morphometric and morphographic variety.

In what concerns the digital model of the studied area, we mention the fact that we had at our disposal 3 (three) categories of data sources: LiDAR, Aster and SRTM 90 (version 4, with a plan resolution of 3 seconds of arc, meaning horizontal accuracy of about 60 meters and vertical of 16 meters), which allowed us to make some DEMs (Digital Elevation Models) with spatial resolution of 5 meters, 30 meters and 90 meters (figure 2).
In order to obtain curvature rasters we used the DTM derived from the SRTM altimetric data, and this because we wanted the quantitative material on which the analysis was done to be drawn out from the elevation models with a large scale of use in geomorphometry. Furthermore, as we already said, our goal is to point the differentiations as concerns the way of interpreting results according to the utilized GIS techniques and not to show the errors of the results arisen in the parameter elaboration due to the DEM accuracy.

**METHODS, RESULTS AND DISCUSSION**

In view of these facts, we consider necessary a short introduction in the mathematical apparatus of the methodologies that underlie the extraction and elaboration functions of the morphometric variable which was brought up here.

In treatises, we can identify two major directions of approach regarding the count way of curvature values: the one of Evans (1980) and the one worked out by Zevenbergen and Thorne (1987). Both methodologies rely, for getting the curvature in one point, on the altitudinal values from the neighbouring cells of the targeted point. We are dealing with a sliding 3 x 3 window in raster (figure 3).
Evans’s method, adjusted by Florinsky (1988), operates with polynominal equations that have 6 parameters, while Zevenbergen and Thorne use 9 polynominal parameters (Jenness, 2012), details are to be found in the tables below (table 1 and 2).

Table 1. Evans’s equation and parameters

<table>
<thead>
<tr>
<th>Z₁₋₉ – altitude values; w – grid cell dimension;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Z</strong> = ( \frac{r^2 + y^2}{2} + zxy + px + qy + w )</td>
</tr>
<tr>
<td><strong>r</strong> = ( \frac{[Z_6 + Z_3 + Z_4 + Z_6 + Z_7 + Z_2 - 2(Z_3 + Z_4 + Z_5)]}{3w^2} )</td>
</tr>
<tr>
<td><strong>t</strong> = ( \frac{[Z_6 + Z_3 + Z_4 + Z_6 + Z_7 + Z_2 - 2(Z_3 + Z_4 + Z_5)]}{3w^2} )</td>
</tr>
<tr>
<td><strong>s</strong> = ( \frac{[Z_6 + Z_3 + Z_4 + Z_6 + Z_7 + Z_2 - 2(Z_3 + Z_4 + Z_5)]}{3w^2} )</td>
</tr>
<tr>
<td><strong>σ</strong> = ( \frac{Z_4 + Z_5 + Z_6 + Z_7 - 2Z_3 - Z_2 - Z_4 - Z_5}{6w} )</td>
</tr>
</tbody>
</table>

Table 2. Evans’s equation and parameters

<table>
<thead>
<tr>
<th>Z₁₋₉ – altitude values; L – grid cell dimension;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Z</strong> = ( A x^2 y^4 + B x^2 y + C x y^2 + D x^2 + E y^2 + F x y + G x + H y + I )</td>
</tr>
<tr>
<td><strong>A</strong> = ( \frac{[(Z_1 + Z_2 + Z_3 + Z_4) - 2Z_2 + 2Z_3 + 2Z_4 + 2Z_5]}{L^2} )</td>
</tr>
<tr>
<td><strong>B</strong> = ( \frac{[(Z_1 + Z_2 + Z_3 - Z_4) - 2Z_2 + 2Z_3 + 2Z_4 + 2Z_5]}{L^2} )</td>
</tr>
<tr>
<td><strong>C</strong> = ( \frac{[(Z_1 + Z_2 - Z_3) - 2Z_2 + 2Z_3 + 2Z_4 + 2Z_5]}{L^2} )</td>
</tr>
<tr>
<td><strong>D</strong> = ( \frac{[(Z_1 + Z_2 + Z_3 - Z_4) - 2Z_2 + 2Z_3 + 2Z_4 + 2Z_5]}{L^2} )</td>
</tr>
<tr>
<td><strong>E</strong> = ( \frac{[(Z_1 + Z_2 - Z_3) - 2Z_2 + 2Z_3 + 2Z_4 + 2Z_5]}{L^2} )</td>
</tr>
<tr>
<td><strong>F</strong> = ( \frac{[-Z_4 + Z_5 + Z_6 - Z_7 - Z_8]}{4L^2} )</td>
</tr>
<tr>
<td><strong>G</strong> = ( \frac{[-Z_4 + Z_5 + Z_6 - Z_7 - Z_8]}{2L} )</td>
</tr>
<tr>
<td><strong>H</strong> = ( \frac{[-Z_4 + Z_5 + Z_6 - Z_7 - Z_8]}{2L} )</td>
</tr>
</tbody>
</table>

Types of curvatures, algorithms, results and discussion

**The profile curvature**

Describe the changing rate of the slope on the versant profile direction, respective along the flow alignments, perpendicular to the level curves (figure 4). To put it otherwise, it indicates the slope variation in vertical plan (Smith et al., 2012). Shary (1995) and Florinsky (1998) call it vertical curvature. In order to get the rasters of this parametre we used two commercial programmes (ArcGIS 9.3, Surfer 9), three open source programmes (Microdem, Landserf, SAGA) and an extension for ArcGIS ("DEM surface tools") devised by Jenness (with the last improvements in 2012). These applications are also used with the rest of the comparative analyses, as far as they have or don’t have implemented functions for the extraction of the targeted types of curvatures.
The main geomorphological significance of the profile curvature is that it indicates the convex, concave and horizontal character of the surfaces, and hydrodynamic with its help we can identify the areas with accelerated outflow and the ones with decelerated outflow of the water on the slopes. The confusions that may arise are connected with the correct ascertainment of the correspondence between the sign of the obtained values and the type of surface associated with it.

In three of the five programmes, the negative values tally with the convex surfaces and in two the negative values tally with the concave surfaces (table 3). Jenness’s extension is designed, as much as possible to coincide with the results obtained in ArcGIS, in terms of the correspondence sign-type of surface, even if the authoress uses Florinsky’s algorithms (multiplies the values with 100 and changes the sign in front of the equations with the ones from „Spatial Analyst”).

The obtained results are more or less surprising, since it was normal that the programmes which contain algorithms that come from the same methodology to give similar values, at least under the aspect of interpretation.

Thus, ArcGIS and SAGA hold functions for the altitude derivatives based on the mathematical expressions of Zevenbergen and Thorne (SAGA offers the possibility of choice from seven functions, but the user manual recommends „Quadratic Surface Method”).

Table 3. The correspondence between the sign of the profile curvature values and type of surface

<table>
<thead>
<tr>
<th>Programme</th>
<th>ArcGIS</th>
<th>Jenness</th>
<th>Surfer</th>
<th>Microdem</th>
<th>Landserf</th>
<th>SAGA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign values</td>
<td>(-)</td>
<td>(+)</td>
<td>(-)</td>
<td>(+)</td>
<td>(-)</td>
<td>(+)</td>
</tr>
<tr>
<td>Corresponding type of surface</td>
<td>cx</td>
<td>ccv</td>
<td>cx</td>
<td>ccv</td>
<td>cx</td>
<td>ccv</td>
</tr>
<tr>
<td>Statistical data: minimum</td>
<td>- 0,704</td>
<td>- 0,656</td>
<td>- 0,0070</td>
<td>-0,664</td>
<td>- 1,718</td>
<td>- 0,0092</td>
</tr>
<tr>
<td>maximum</td>
<td>+ 0,927</td>
<td>+ 0,873</td>
<td>+ 0,0092</td>
<td>+ 0,798</td>
<td>+ 1,224</td>
<td>+ 0,007</td>
</tr>
<tr>
<td>std dev.</td>
<td>0,109</td>
<td>0,096</td>
<td>0,0010</td>
<td>0,109</td>
<td>0,18</td>
<td>0,001</td>
</tr>
</tbody>
</table>
The algorithm for the profile curvature is the following:

$$K_{pr} = \frac{-2(DG^2 + EH^2 + FGH)}{G^2 + H^2}$$

Putting aside the huge numerical differences (table 3), explainable by the fact that in ArcGIS the real results are multiplied with 100, as Zevenbergen and Thorne recommends (1987), in order to be easier manipulated and reclassified, we observe that the interpretations are completely reversed: in ArcGIS the positive values indicate concave surfaces and a deceleration of the water flow on the slopes, while the negative values reflect the convex surfaces and consequently an acceleration of the water flow (ArcGIS Desktop 9.3 Help, 2011), the 0 (zero) value being equivalent with the horizontal surfaces, and in SAGA the interpretation is reversed (figure 5). The only logical explanation comes from Smith et al. (2012), who says that the result is multiplying with \([-100]\) and due to this, the equation would receive in its front the positive sign.

Figure 5. Profile curvature: a – ArcGIS; b – SAGA
Surfer Programme uses extraction functions for the curvature values, whatever type, based on the equations taken from Mitasova and Hofierka (Surfer User’s Guide, 2002), respectively Moore et al. (1991). The algorithm for the profile curvature is:

\[
K_p = \frac{\frac{\partial^2 z}{\partial x^2} \left(\frac{\partial^2 z}{\partial x^2}\right)^2 + 2 \frac{\partial^2 z}{\partial x \partial y} \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} + \frac{\partial^2 z}{\partial y^2} \left(\frac{\partial z}{\partial y}\right)^2}{q^{1/4}}
\]

\[
p = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 ; \quad q = 1 + p
\]

The meaning of the results is similar to the one from ArcGIS, that is the negative values show convex surfaces, and the positive ones indicate concave areas, so we will not insist upon them. Remarkable are instead the numerical results obtained in the two programmes: if we ignore all about the multiplication with 100 from ArcGIS, the given values, about the maximum and minimum, are identical. From this point of view Schmidt et al. (2003), placing his reliance on a personal communication from Florinsky, affirms that Zevenbergen - Thorne and Moore’s algorithms are the same, concerning the chosen mathematical model, and the observed differences are justified by the different way of using the symbols (differences in notation).

Landserf and Microdem use similar equations for curvatures, in the sense that both are based on Evans method. Microdem takes Wood’s algorithms (who designed Landserf application within his thesis, in 1996) for the minimum and maximum curvature, and for the profile and plan curvature he keeps unchanged Evans’s equations, while in Landserf they are slightly modified by Wood. Hence the differences regarding the interpretation of values from table 3.

**Plan curvature**

It can be specified depending on the slope of the surface or on the display. If we relate to the slope, it reflects the changing rate of the slope on a parallel direction with the isohypses. In the second situation, we say that it shows the display gradient along the intersection line between the terrestrial surface and the XY plan. Schmidt (2003) calls it horizontal curvature or contour curvature. Its significance is the following: it offers information regarding the convergent and divergent character of the flow.

In what concerns the correspondence between the sign of the obtained values and the flow character, these can be found in table 4. In ArcGIS, SAGA and Microdem the negative values indicate convergent flow, and the positive values match the surfaces with divergent flow, while in Surfer and Microderm the correspondence is reversed.

**Table 4.** The correspondence between the sign of the profile curvature values and type of flow.

<table>
<thead>
<tr>
<th>Programme</th>
<th>ArcGIS</th>
<th>Surfer</th>
<th>Microdem</th>
<th>Landserf</th>
<th>SAGA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign values</td>
<td>(-)</td>
<td>(+)</td>
<td>(-)</td>
<td>(+)</td>
<td>(-)</td>
</tr>
<tr>
<td>Corresponding type of flow</td>
<td>conv.</td>
<td>div.</td>
<td>div.</td>
<td>conv.</td>
<td>div.</td>
</tr>
<tr>
<td>Statistical data: minimum</td>
<td>-0.7989</td>
<td>-0.0069</td>
<td>-0.66</td>
<td>-10</td>
<td>-0.0079</td>
</tr>
<tr>
<td>maximum</td>
<td>+0.6649</td>
<td>+0.0091</td>
<td>+0.79</td>
<td>+10</td>
<td>+0.0066</td>
</tr>
<tr>
<td>std dev.</td>
<td>0.109</td>
<td>0.021</td>
<td>0.109</td>
<td>2.2976</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

We mention the fact that the algorithm used in ArcGIS and SAGA for the plan curvature (Zevenbergen & Thorne, 1897), is in fact the mathematical expression for the cross – sectional curvature (Jenness, 2012):
Surfer has implemented the algorithm taken from the authors mentioned at the profile curvature:

$$K_{pl} = \frac{2(DH^2 + EG^2 - FGH)}{G^2 + H^2}$$

This explains the differences in the interpretation of results regarding the three programmes. For illustration, we present comparatively, in raster format, the plan curvature obtained in Arc and Surfer (figure 6). Regarding Microdem and Landserf, the differences are induced by the changes made by Wood within the mathematical expressions of Evans.

**Figure 6.** Plan curvature: a – ArcGIS; b – Surfer
**Tangential curvature**

It was introduced by Krcho (1991), Mitasova and Hofierka (1993), having the same significance as the plan curvature (the curvature measured on a perpendicular alignment on the direction of the highest slope), but by the way of calculation (the value of the plan curvature is multiplying with the sine slope), the obtained values highlights better the differentiations in the land. Besides, the authors quoted for this type of curvature, recommend that it should be used in the place of the plan curvature, especially in the studies that consider the characteristics of the flow on particular surfaces.

The programme that has implemented the algorithm for the tangential curvature is Surfer:

\[
K_{tg} = \frac{\frac{\partial^2 z}{\partial x^2} (\frac{\partial z}{\partial x})^2 - 2 \frac{\partial^2 z}{\partial x \partial y} \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} + \frac{\partial^2 z}{\partial y^2} (\frac{\partial z}{\partial y})^2}{pq^{\frac{3}{2}}} = K_{pl} \left( \frac{p}{q} \right)^{\frac{1}{2}}
\]

\[
p = \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2 ; q = 1 + p
\]

The negative values indicate the areas with divergent flow, and the positive ones match the surfaces with convergent flow. It can also be obtained with Jenness toolset for ArcGIS, but the interpretation of the sign values is reversed (it matches with the plan curvature interpretation from the respective programme).

**Longitudinal and Cross-sectional Curvature**

From a conceptual point of view they are similar to the profile curvature, respectively to the plan curvature, also having the same geomorphologic and hydrodynamic interpretation.

Landserf has incorporated algorithms for the both parameters, and Microdem has only for the cross-sectional curvature, being designed by Wood (1996):

\[
K_{lon} = \frac{-200(a d^2 + b e^2 + c de)}{(a^2 + b^2)}
\]

\[
K_{cr} = \frac{-200(b d^2 + a e^2 - cde)}{(a^2 + b^2)}
\]

In the extension devised by Jenness, these curvature varieties correspond in reality to the expressions of the plan and profile curvature from ArcGIS – „Spatial Analyst Tools”.

**Mean, maximum and minimum curvature**

The mean curvature is mostly specified depending on the maximum and minimum curvature (Smith et al., 2012):

\[
K_{med} = \frac{(K_{max} + K_{min})}{2}
\]

There are also situations when it is considered as difference between the profile curvature and the plan curvature or as mean value of the curvature for each grid cell. Olaya (2009) says that this parameter is important especially in the geomorphologic researches, because it expresses the medial values of the landform convexity and concavity. The equations for the extraction of the three varieties of curvatures are found in Landserf, being developed by Wood (1996), on Evans’s method (1980).

**General curvature**

It was identified by Moore and co. (1993), and in a different form by Zaslavsky and Sinai (1981).

It is considered to be a general measure of the land convexity, in the sense that the positive values indicate the convex surfaces that is the interfluves and the peaks, and the negative values
match to the concave surfaces, that is the valleys and the cavities. The zero value is afferent to the horizontal surfaces (figure 7). These correlations are valid for the ArcGIS and SAGA algorithm: 

$$c_p = -20\phi + \phi$$

as Moore uses the formula without the negative sign in front of it, which reverses the significance of the values.

CONCLUSIONS

Out of the conducted analyses we observe that the differentiations concerning the results and the qualitative interpretation of the curvature values are caused by:

- the type of the used mathematical model and the adaptations of the functions for extracting the parameters, adaptations integrated in the GIS softs that we used;
- the [-] or [+] sign placed in front of the algorithms and the multiplication with 100 in the case of some programmes (ArcGIS, Landserf, Microdem);
- the confusions that appear regarding the implemented algorithm within a programme and the name, respectively the definition of the curvature type processed on it; for example, the profile curvature from ArcGIS is actually the longitudinal curvature (Jennesse, 2012, who takes the information from Shary and Florinsky);

Geomorphological, we consider that a special significance have the following curvatures: the profile curvature, the tangential curvature and the general curvature.

Therefore, in that they offer the possibility to identify the convex/concave and horizontal areas, respectively the ones with convergent and divergent flow, they permit the appraisal of the potential water erosion organised areolar and linear on the flanks (gully erosion). Besides, Mitas and Mitasova (1998) have designed a model to estimate the potential of erosion/of sedimentary deposit based on the profile and tangential curvature. We also mention the fact that the values of the profile curvature allow the appraisal of the stage of relief, because, obviously, the profile...
aspect of the flanks (their shape), correlated with elements of lithology, geological structure, vegetation and anthropogenic insertion, assert the dominant types of geomorphologic processes and their geomorphologic productivity in time.

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